

Statement of Interest

Matias Relyea

My name is Matias Relyea, and I am a freshman at the University of North Carolina at Chapel Hill pursuing a B.S. in Mathematics. I am applying to the Duluth REU because I would like the rare opportunity to enrich my study of pure mathematics from my existing background in algebraic number theory, and I believe Duluth's dual prioritization of serious mathematics and fun is something that will aid me in becoming a more human mathematician. I am also strongly interested in number theory, and Duluth will provide me with a robust research opportunity in number theory that UNC cannot provide, as there are not many faculty who work in that field.

The first time that I truly realized that math was what I wanted to do was when I took a January term mini-course on surreal numbers in 2024. At the time, I had already been enthralled by number theory, having done quite a bit of number theory reading through lecture notes. Surreal numbers, however, opened a new door to John Conway's *On Numbers and Games*, and I couldn't get enough of its whimsy and uselessness. A YouTube video by Owen Maitzen on Hackenbush was just so entertaining, and this inspired me to work on omnific integers, a number-theoretic connection with surreal numbers that Conway had minimally mentioned in his book. I gave a short presentation on integer clouds and some new combinatorial observations regarding omnific integers; this was the entirety of the January term, but I managed to get my hands on a mini copy of Donald Knuth's *Surreal Numbers* and I look back fondly on the experience.

This was a beginning point for my interest in serious research. My background is largely in number theory, though, like the January term on surreal numbers, I have developed interests in combinatorics and algebra. My first introduction to formal math and research was the Independent Research and Paper Writing class at Euler Circle in the summer of 2022, where I wrote a 44-page paper on quadratic reciprocity and its applications. I presented a 15-minute talk at its conclusion. While most of my investigations were in elementary number theory, mostly pertaining to Eisenstein's lattice-counting proof of the famous result, my paper also covered quadratic Gauss sums, beginning a close interest in algebraic number theory and higher reciprocity in the following years.

I developed this newfound interest by reading Rosen and Ireland's *A Classical Introduction to Modern Number Theory* under the guidance of Professor Tamar Avineri, who taught an illuminating elementary number theory course at the North Carolina School of Science and Mathematics. Supplementing this was Joseph Gallian's *Contemporary Abstract Algebra*, where I gained a great understanding of group and ring theory, as well as facts about residue class rings that I would need later. Professor Avineri's mentorship culminated in

two papers, with *The Quadratic and Cubic Characters of 2* being published in *Mathematics Magazine* (2026), and *On Finite Fields and Higher Reciprocity* being presented at the NCSSM Research Symposium 2024. One particular aspect that excited me about working on the *Mathematics Magazine* paper was being able to learn about the history of mathematics (as I experienced thoroughly through Lemmermeyer's *Reciprocity Laws: From Euler to Eisenstein*), and solving small problems that I had never even thought to consider on the way. Unfortunately, Professor Avineri passed away in May of 2025, though her impact on my approach to exposition and research was undoubtedly immense, and my love for number theory and desire to pursue graduate study in mathematics was solidified by her guidance.

In my second January term, I collaborated with two classmates on an original discrete math research project concerning polyomino placement in colorings of square and triangular grids during the following January term. This experience allowed me to get a taste of research mathematics in a fast-paced collaborative setting that demanded constant original contribution. Working intensively over two weeks resulted in a paper that was ultimately accepted to the NCSSM scientific journal, *Broad Street Scientific*. This paper also culminated in a presentation at the NCSSM Research Symposium 2025.

At UNC, I have pursued coursework in analysis and advanced linear algebra, served as an undergraduate learning assistant for a discrete mathematics course, and completed a directed reading program on Tom Leinster's *Basic Category Theory*. Adjusting to advanced proof-based coursework presented challenges in my first semester at UNC, but this experience has strengthened my mathematical maturity and approach to problem-solving. In the spring, I am taking topology, algebraic geometry, and algebraic structures. I am also preparing two one-hour talks on the contents of my paper published in *Mathematics Magazine* for presentation to the university math club.

Altogether, I believe Duluth will be an excellent place to develop my skills in algebraic number theory further while introducing me to a community of like-minded and passionate mathematicians. I plan on pursuing a PhD in mathematics, and Duluth will simultaneously provide me with something fun to do over the summer and give me an opportunity to work on an exciting project in number theory.